Valuation of Washington DNR Timber Harvest Contracts under Price Uncertainty
A Real Options Approach

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Outline

1. Concepts
   - Contract Specifics
   - Valuation

2. Example
   - Assumptions
   - Results
Washington DNR Timber Harvest Contracts

Brief Overview

- **Types:**
  - **Lump-Sum Sale:** All timber sold to highest bidder for cash.
  - **Scale Sale:** All timber sold to highest bidder by volume or weight.
  - **Contract-Harvest Sale:** Log sorts from a stand sold individually.

- **Properties:**
  - Fixed contract length
  - Harvest any time prior to contract expiry
  - Uncertainty about future timber prices

- Fees omitted for clarity.
American Call Options

Basic Properties

- Derivative contracts commonly traded in financial markets.
- Characteristic Properties:
  - **Length**: Fixed at time of sale.
  - **Strike Price**: Fixed at time of sale.
  - **Number of Shares**: Fixed at time of sale.
  - **Call Property**: The right, but not the obligation, to purchase an asset at strike price.
  - **American Property**: Early exercise any time prior to expiry date.
Timber Harvest Contract as Real Option

- Timber harvest contract is an American call option on the value of timber.

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<th>American Call</th>
<th>Harvest Contract</th>
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<td>Fixed Harvest Cost</td>
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<td>Strike Price</td>
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- The forestry firm has the right to purchase the timber at the harvest cost and sell it in the market at the prevailing price.

- Techniques for valuation of financial options can be adapted to valuation of timber harvest contracts.
American Option Valuation

- Early exercise possibility greatly complicates American option valuation.
- Closed-form solutions of problems pertaining to valuation American options are limited to special cases.
- Solutions approximated by the use of numerical methods.
- Valuation of American options is an area of active research.
- A popular technique is the LSM algorithm introduced by Longstaff and Schwartz in 2001.
At each point in time, prior to harvesting, the forest owner must make a decision:

\[
\max \left[ CF_t, \mathbb{E} \left( \sum_{t+1}^{T} d_i \, CF_i \, \middle| P_t, P_{t-1}, \ldots \right) \right]. 
\]

(1)

i.e. Choose the larger of:

- Profit from harvesting today, \( CF_t = Q_t (P_t - C) \), or
- Discounted expected profit from harvesting in the future, \( \mathbb{E} \left( \sum d_i \, CF_i \right. \ldots \) 

The conditional expectation term is critical to contract valuation yet can rarely be calculated exactly.
The LSM algorithm provides an estimate of the \( E \left( \sum d_i CF_i \mid \ldots \right) \) term in equation 1.

The LSM starts at time \( T \) and moves backward in time:

1. Generate price vectors \( P_t \) and \( P_{t+1} \).
2. Use \( P_{t+1} \) to calculate \( CF_{t+1} \) a vector of cashflows from harvesting at time \( t+1 \).
3. Calculate \( \hat{E} (\cdot | \cdot) \) by regressing \( CF_{t+1} \) on a set of basis functions of \( P_t \).
4. Substitute \( \hat{E} (\cdot | \cdot) \) into equation 1 and decide whether to harvest at time \( t \) or not. Update record of harvest values accordingly.
5. Repeat previous steps until reaching contract starting date.
Many models for price behavior are available. Choice is driven by available historical data and economic theory. The model used for this example is a log mean-reverting stochastic process:

\[ dS_t = \kappa (\mu - \ln S_t) S_t \, dt + \sigma S_t \, d\mathbb{W}_t, \]  

(2)

where \( \kappa \) is the rate of mean reversion, \( \mu \) is the log of long term price, \( \sigma \) represents price volatility and \( d\mathbb{W}_t \) is an increment of the Wiener process.
Parameter Settings

- **Harvest Cost** equal to $100 per MBF.
- **Rate of Discount** assumed to be 5% per year.
- **Yield Function** assumed high-yield site in Western Washington.
- **Initial Stand Age** equal to 50 years.
- **Contract Length** equal to 3 years.
- **SDE Parameters**: long-term trend $\mu = 6.11$, rate of reversion $\kappa = 0.40$, volatility $\sigma = 0.25$
- **Initial Price** equal to $450 per MBF.
Timber Price Volatility

Contract Value as a Function of Timber Price Volatility
The Washington DNR timber harvest contract is, in essence, an American call option on the value of timber.

An enhanced form of the LSM algorithm provides an effective tool for timber harvest contract valuation.

Outlook
- Replication of the results for a variety of stochastic price models.
- Application to a large set of past contracts.
References I
